

# **New methods for evaluating one-loop corrections of multi-jet production at the Large Hadron Collider**

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